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How Some Infinities Cause Problems in Classical Physical Theories

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Abstract

In this paper we review a 1992 excursion of Jean Paul Van Bendegem into physics, ‘How Infinities Cause Problems in Classical Physical Theories’, in the light of two later models concerning colliding balls, of Pérez Laraudogoitia and of Alper and Bridger, respectively. We show that Van Bendegem anticipated the model of Alper and Bridger by six years, but we also argue that his conditions for the avoidance of problems in these models are not entirely adequate. For although a veto on actual infinity seems to be required, allowing a potential infinity remains a viable option.

Keywords: Zeno balls, finitism, potential infinity.

1 Finitism in Classical Mechanics

We all know it: Jean Paul Van Bendegem does not like infinity. In his view, both mathematics and physics can do extremely well without the concept of an actual, or even of a potential infinity. And although he realizes that there is a downside to relegating infinity to the scrapheap of mathematical history, he has always insisted that the advantages outweigh the drawbacks.

Far be it from us to question the feasibility of Jean Paul’s program: his ideas contain many valuable insights that should be taken seriously. We do not challenge the consistency of the strict finitist reconstruction of large parts of mathematics; nor have we the temerity to deny that infinity has given rise to many a headache for the physicist. Indeed, we concur with his claim that:

“...infinities cause all sorts of bizarre problems in the framework of classical mechanics.” (Van Bendegem 1992, p. 33).

Yet we will argue that, in classical mechanics, only actual infinities give rise to difficulties, while potential infinities are not only harmless, but in fact quite useful. That explains the title of our paper, which differs by only one word from the title of Van Bendegem’s cited 1992 paper, ‘How Infinities Cause Problems in Classical Physical Theories’.

One of the headaches of the purveyor of classical mechanics is that determinism can fail if infinities are allowed to creep into the theory. In fact it is claimed on page 35 of the cited paper that determinism in classical mechanics *will* fail if any *one* of the following conditions is violated:

- (C1) No forces may be infinite
- (C2) No masses may be infinite

(C3) No accelerations may be infinite

(C4) No velocities may be infinite

Interestingly, much of what Jean Paul says in this paper anticipates a completely independent assault on determinism which started in 1996 and has continued to the present day. We mean the assault launched by J. Pérez Laraudogoitia in 1996 and further developed by J. Alper and M. Bridger in 1998. We propose to describe first Pérez Laraudogoitia's infinitistic model of colliding balls; and then we shall confront it with Van Bendegem's conditions (C1)–(C4). Next we discuss the reaction to this work by Alper and Bridger. The essence of Alper and Bridger's model was already put forward by Van Bendegem in 1992; we shall however argue that neither his resolution of the looming paradox, nor that of the later authors, is entirely satisfactory.

2 Zeno balls

In 1996 the Basque philosopher Pérez Laraudogoitia published a paper in *Mind* with the modest title: 'A Beautiful Supertask' (Pérez Laraudogoitia 1996). In it he single-handedly demolished the laws of conservation of energy and momentum, and moreover he breached that bastion of Newtonian mechanics, determinism. We propose to examine his model, and to see which of Van Bendegem's conditions (C1)–(C4) has been contravened.

In general, any system containing a finite number of balls that undergo a finite number of elastic collisions amongst themselves respects the laws of conservation of energy and momentum. However, consider the following idealized system. An infinite number of identical point masses (balls) are placed at the Zeno points $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \dots$ on a straight line. All the balls are at rest except the first, at 1, which moves with constant speed towards the second, at $\frac{1}{2}$ (see Fig. 1). After elastic collision the first ball comes to rest, passing all its kinetic energy on to the second ball, which soon collides with the third ball, which acquires all the energy, and so on *ad infinitum*. However, after the *finite* time that it would have taken the first ball to reach the point 0, had the other balls not been in its way, every ball will have moved briefly, but then have been brought to rest. After all motion has subsided, the energy and momentum of the balls have disappeared without trace.

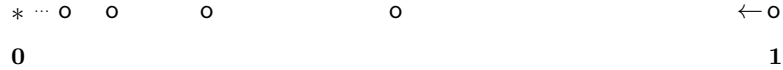


Figure 1. Collision of an infinite number of identical balls

The conclusion is that the laws of conservation of energy and momentum have been violated. What is more, since classical mechanics is time-reversal invariant, a video recording of the above scenario of collisions, run backwards in time, should also depict a possible mechanical evolution that is consistent with the Newtonian equations. Such an evolution begins with an infinite number of identical balls that are at rest. At a certain moment motion arises spontaneously, out of the origin, and balls move to the right, successively passing on the motion to their rightmost neighbour until the last ball carries off all the energy and momentum. Since the moment at which the motion commences, and its magnitude for that matter, are arbitrary, the system is grossly indeterministic.

What has the Belgian to say to the Basque? Since there are infinitely many balls, all of the same mass, the total mass of all the balls is infinite, in clear violation of condition (C2), so

the breakdown of determinism is no surprise. It does not help to remove the restriction that the balls be point masses. For suppose the balls, instead of being point masses, are spheres of geometrically decreasing radii, fitting on a finite line segment (see Figure 2). Then, if the balls are progressively more dense, in such a way that they all have the same mass, the analysis goes through unchanged: an infinite number of elastic collisions leads to the loss of all energy and momentum, and to the breakdown of determinism.

How would it be if the density of the progressively smaller balls were *constant*, so that the masses decrease geometrically? Now each ball is not brought to full rest by collision with its neighbour, so that it retains some kinetic energy after its final collision. Is it possible that the sum of the energies of all the balls, after the infinite sequence of collisions has taken its course, is equal to the initial energy? Indeed that is what happens when the masses decrease in geometrical progression, at any rate if the rules of Newtonian mechanics are followed (Atkinson and Johnson 2009). Energy and momentum are conserved, determinism is inviolate, and sanity seems to have been restored.

It turns out that the velocities of the balls are not bounded from above: some of the very tiny balls, indeed all but a finite number of them, acquire speeds in excess of that of light. Since the total mass of all the balls is now finite, condition (C2) is respected by the new system of balls, but how about (C4)? The velocities have now no upper bound, but none of them is actually infinite. Van Bendegem writes that his prohibition (C1)–(C4)

“... does not imply that masses, forces, accelerations and velocities have finite upper bounds. It may very well be that arbitrarily large values are allowed, only excluding the occurrence of the infinite value itself.” (Van Bendegem 1992, p. 33).

Apparently, then, the unboundedness of the velocities does not count as a violation of (C4).

Superluminal velocities were anathema to Albert Einstein, if not to Isaac Newton. What happens if we reconsider the Zeno ball system, with geometrically decreasing masses, according to special relativistic mechanics? When this is done, it is found that the tiny balls have speeds close to, but always less than that of light, as expected. One might also expect the system to obey the law of conservation of energy-momentum, but it does not!

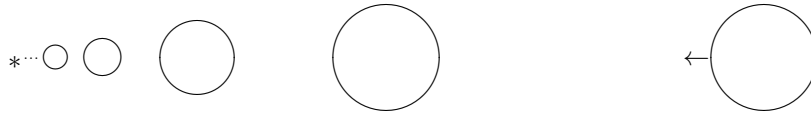


Figure 2. Collision of an infinite number of progressively smaller balls

When the energy-momenta of the balls are added up, after all the collisions have taken place, it is found that some energy and some momentum has been lost. Although energy-momentum is conserved after a finite number of elastic collisions, after an infinite number of them this is no longer the case. Moreover determinism has been thrown out of the window too, for the time-reversed scenario involves once more an undetermined time at which energy-momentum is created *ex nihilo* at the origin (Atkinson and Johnson 2009).

How do Van Bendegem’s conditions fare in this relativistic setting? (C4) is safe, for all velocities are less than that of light, although they do approach that finite value asymptotically. (C2) is also inviolate, for the sum of the rest masses is finite. In fact the sum of the kinetic masses is finite too, for it turns out that this sum can never exceed the initial energy of the

first ball at the beginning of the exercise. How about (C1) and (C3)? Are there any infinite forces or accelerations? In a sense there are, if we stick to what are usually called impulsive forces; but this is surely a red herring. We can easily replace the impulsive forces of collision by finite-range forces (for example, where the range decreases along the line of Zeno balls in such a way that it is never greater than one-third of the initial distance between successive balls), and if we do that, (C1) and (C3) are clearly maintained.

What does this mean? We have managed to respect (C1)–(C4), and yet determinism has fallen by the wayside! Van Bendegem writes

“Summarizing, what these few examples clearly show is that (C1)–(C4) really are necessary to avoid the conclusion of indeterminism. However, it does not show that the subset of models that satisfies these four principles will coincide with the deterministic models. In other words, what guarantee do we have that indeed all these models are deterministic?” (Van Bendegem 1992, pp. 42–43).

So no watertight claim is being made that (C1)–(C4) are sufficient to guarantee determinism, but we do read

“Although I have not been able — in fact, I am quite skeptical ... that it could be possible at all — to find a formal proof that (C1)–(C4) are sufficient, I did find additional arguments in its favour.” (Van Bendegem 1992, p. 43).

We submit that the search might as well be stopped: (C1)–(C4) are demonstrably insufficient to guarantee determinism, for our relativistic Zeno ball model respects them all, but is indeterministic. We have seen that for Jean Paul each of the conditions (C1)–(C4) is necessary for determinism, thus two questions suggest themselves:

1. Was it essential to employ Einstein’s mechanics to show that (C1)–(C4) are insufficient? Can the same thing be done using Newton’s mechanics?
2. What must be added to (C1)–(C4) to achieve necessary *and sufficient* conditions for determinism?

As to the first question, we can be brief. Although determinism is respected in classical mechanics when the masses of the balls decrease geometrically, that is not so when the masses decrease more slowly. For example, if the n th ball has a mass that is proportional to $1/n^2$, it can be shown that: (1) the total mass of all the balls is finite, thus safeguarding (C2); (2) energy is not conserved by the infinite set of Zeno balls; and (3) indeterminism reigns (Atkinson and Johnson 2009). (C4) is not violated, since no ball has infinite velocity, even though there is no upper bound on the velocities. The finitude of forces and accelerations can again be assured by the subterfuge of finite-range, instead of impulsive forces. Thus (C1)–(C4) are insufficient as guarantors of determinism, also in classical (Newtonian) mechanics.

The quest for sufficient conditions to assure determinism is more subtle: we will consider the matter at length in the next section.

3 Colliding with an open set

In this section we consider an enrichment of the Zeno ball model in which Van Bendegem anticipated — by six years — a model that Alper and Bridger used to criticize Pérez Laraudogoitia’s paper (Alper and Bridger 1998).

As in Figure 1, an infinite number of identical, stationary Zeno balls are placed at the Zeno points $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \dots$ on a straight line. Van Bendegem took the balls to be of decreasing size and mass, but, as he points out, the same point can be made with identical point masses, so we

will first present the argument in those terms. The novel feature is that a further ball, which we shall call the *Van Bendegem ball*, or VB ball for short, is identical to the others and is situated on the line to the left of the origin, 0. It moves with constant speed towards the origin (see Fig. 3).



Figure 3. VB ball and Zeno balls

There is no ball at 0, but the origin is a point of accumulation of the locations of the Zeno balls. If the VB ball were to collide with a Zeno ball, it would come to rest, thereby imparting all its energy to the Zeno ball, which would move off with the speed that the VB ball originally had. However, there is no Zeno ball with which it could collide. For suppose, *per impossibile*, that it did collide with one of the Zeno balls. Then it should first have collided with that Zeno ball's immediate left-hand neighbour, and this would have brought it to rest, making the posited collision impossible. Thus the VB ball can collide with none of the Zeno balls. In the absence of any forces other than those arising from collision, the VB ball must continue in its state of constant motion (Newton's first law), thus arriving in a finite time at the Zeno point 1. But this is also impossible, since an infinite number of Zeno balls should have blocked its way.

Such is the scenario sketched by Van Bendegem, and later by Alper and Bridger. The latter authors conclude that what we have called the VB ball must simply cease to exist when it arrives at the origin, since it can neither be stopped there, nor can it progress further, nor indeed can it be anywhere else. Not only has energy disappeared without trace, as in Pérez Laraudogoitia's related problem, but the ball itself has vanished into thin air!

However, Alper and Bridger's system constitutes a logical contradiction. To be precise the following conditions are inconsistent with one another:¹

1. Stationary balls of unit mass and zero size – mass points – are situated at the Zeno points.
2. A moving ball of unit mass and zero size travels at constant speed, reaching 0 from the left.
3. When the moving mass point occupies the same position as a stationary mass point, it comes to rest, otherwise it continues in its state of constant motion.
4. The moving ball comes to rest before reaching the point 1.

While it is formally possible in classical logic to deduce anything from a contradiction, of course to say anything significant about a physical system (however idealized) one should start from a noncontradictory set of statements. Moreover, Alper and Bridger's statements are doubly suspect, for they ask us to believe that there is no trouble until the VB ball reaches the origin, but that the system becomes paradoxical at the moment that the ball reaches that point. However, that is a misreading of the situation: the system as described by the four above conditions is inconsistent *tout court*, not simply at one particular time.

What did Jean Paul Van Bendegem say about the paradox, six years before Alper and Bridger sunk their teeth into the problem? After noting, as above, that the VB ball cannot

¹This, and other inconsistent systems that are isomorphic to it, have been considered by Peijnenburg and Atkinson (Peijnenburg and Atkinson 2008; Peijnenburg and Atkinson 2010).

collide with any of the Zeno balls, he concludes that the VB ball must move through the Zeno balls, which, as he wryly admits, is

“... an odd conclusion to say the least.” (Van Bendegem 1992, p. 40).

He claims

“The solution to [this] problem is easily seen ... Instead of keeping the [Zeno balls] separate, join them. Instead of spheres, turn them into rectangles with the same height H for each one, but with diminishing length L_i .” (Van Bendegem 1992, pp. 40–41).

As we noted, Van Bendegem is in the first instance talking about balls of finite, decreasing size, rather than point masses. His model is illustrated in Figure 4.

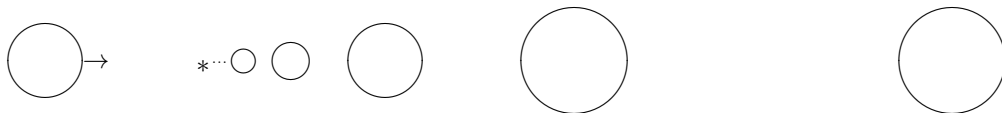


Figure 4. VB ball and an infinite number of progressively smaller balls

Van Bendegem proposes turning the spheres into ‘rectangles’ — or more properly right parallelepipeds — and then joining them to produce one solid body. He continues:

“Let the sum of all these lengths be L . Then it is obvious what we obtain: a nice rectangle of size L by H ... when we are talking about an object ... normally speaking, we include the boundary or limit points.” (Van Bendegem 1992, p. 41).

It seems to us however that the proposed ‘solution’ does not address the Van Bendegem-Alper-Bridger paradox at all. The model in which the balls are all joined up is, as a mathematical problem, quite different from the model depicted in Figures 3 and 4. The former model has a straightforward, deterministic solution, whilst the latter is mechanically impossible. It would not help to add a point mass at the origin, the limit point of the Zeno balls. For although the VB ball could then collide with this extra point mass, the latter could not make contact with any of the Zeno balls, and this for the same reason as before.

Finally, Jean Paul the confessed strict finitist adds:

“Note too that these examples illustrate in a quite clear way that not all infinities in classical mechanics are to be eliminated. Limit points are typically products of an infinite process. If one were to insist on the elimination of all infinities, then the proposed solution would be excluded.” (Van Bendegem 1992, p. 42).

We are not particularly fond of Alper and Bridger’s conclusion that, since the system involves a logically inconsistent set of properties, the ball must spontaneously disappear when it reaches the point of accumulation. A more straightforward way to avoid the inconsistency, in our view, is to ban an actual infinity of balls. If we insist that the number of Zeno balls is only potentially infinite, we can easily answer the question as to what happens to the VB ball. The properties of the (potentially) infinite set of Zeno balls are defined to be the limits (when these exist) of properties of a finite number of balls, as that number increases without bound.

When the Zeno balls are all identical, we find that the VB ball comes to rest. If the masses of the Zeno balls decrease geometrically, the results of numerical calculations show that, when the new ball is light, it rebounds, but if it is sufficiently massive, it either stops, or it merely slows down, i.e. it moves with a reduced positive velocity after all collisions have taken place (Atkinson and Johnson 2010).

In conclusion, we suggest that, in addition to Van Bendegem's four conditions (C1)–(C4) one should add the condition that all actual infinities had better be banned from physics. Indeed, a case could be made that the limitation of mechanics to the potentially infinite provides sufficient means to avoid all the pathological mathematical creatures on Jean Paul Van Bendegem's fascinating roll call.

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